Economics 113 Prof. Ross Starr, Mr. T. Kravitz
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Problem Set 5

It's OK to work together on problem sets.

1. Starr's *General Equilibrium Theory*, problem 7.2.

2. Consider an Edgeworth Box for two households. The two goods are denoted x, y. The households have identical preferences:

$$(x,y) \succ (x',y') \quad \text{if} \quad 3x+y > 3x'+y', \quad \text{or}$$
$$(x,y) \succ (x',y') \quad \text{if} \quad 3x+y = 3x'+y' \quad \text{and} \quad x > x'.$$
$$(x,y) \sim (x',y') \quad \text{only if} \quad (x,y) = (x',y').$$

They have identical endowments of (10, 10). Demonstrate that there is no competitive equilibrium. Is this example a counterexample to Theorem 7.1 (does it demonstrate that Theorem 7.1 is false?) ? Explain.

3. Consider a small economy, with two goods and three households. The two goods are denoted x, y. The households have identical preferences described by the utility function

u(x, y) = sup [x , y] . Where sup indicates the supremum or maximum of the two arguments. Demonstrate that these preferences are nonconvex; they do not fulfill Starr's *General Equilibrium Theory* assumptions C.VI or CVII.

The households have identical endowments of (10, 10). Demonstrate that there is no competitive equilibrium in this economy [Hint: Show that price vector $(^1/_2 + \varepsilon, {}^1/_2 - \varepsilon)$, $\varepsilon > 0$, cannot be an equilibrium; similarly for $(^1/_2 - \varepsilon, {}^1/_2 + \varepsilon)$; and finally $(^1/_2, {}^1/_2)$. That pretty well takes care of it.]

4. Starr's *General Equilibrium Theory* problem 7.6, parts (i), (ii). Part (iii) is rewritten below. "competitive equilibria" means "competitive general equilibria."
(iii) Assuming in addition continuity of $\tilde{Z}(p)$, Q has a fixed point $p^* \in P$ so that $Q(p^*)=p^*$. Does this prove that under these assumptions the economy has a competitive general equilibrium?

5. Let $f:P \rightarrow P$, f continuous. Define $Z(p) = f(p) - \left[\frac{p \cdot f(p)}{p \cdot p}\right] p$. The term in square brackets is just a scalar multiplying the vector p. Show that $p \cdot Z(p) = 0$. Z is a continuous function, $Z:P \rightarrow R^N$. Why? Assume there is a competitive equillibrium price vector p* so that Z(p*) = 0 (the zero vector; ignore excess supplies of free goods) . Is p* also a fixed point of f so that f(p*) = p*? Review Theorem 11.2 in Starr's *General Equilibrium Theory* to see what you've demonstrated.